# Exam 2 Part 2

**Problem 1:**

syms x;

y = 3\*x^5 \* (sin(5\*x))^2 + 100\*exp(-4\*x) \* cos(3\*x);

% Calculate the first derivative

dydx = diff(y, x);

% Calculate the second derivative

d2ydx2 = diff(dydx, x);

% Evaluate the first derivative at x=0.01

x\_val\_1 = 0.01;

dydx\_at\_x\_val\_1 = double(subs(dydx, x, x\_val\_1));

% Evaluate the second derivative at x=0.1

x\_val\_2 = 0.1;

d2ydx2\_at\_x\_val\_2 = double(subs(d2ydx2, x, x\_val\_2));

% Display the results

disp(['First derivative at x=0.01 is: ' num2str(dydx\_at\_x\_val\_1)]);

disp(['Second derivative at x=0.1 is: ' num2str(d2ydx2\_at\_x\_val\_2)]);

First derivative at x=0.01 is: **-392.7887**

Second derivative at x=0.1 is: **923.7175**

**Problem 2:**

syms x;

y = 3\*x^5 \* (sin(5\*x))^2 + 100\*exp(-4\*x) \* cos(3\*x);

% Calculate the definite integral over the interval -1 to 3

integral\_value = int(y, x, -1, 3);

integral\_value = double(integral\_value);

% Display the result

disp(['The value of the integral over the interval -1 to 3 is: ' num2str(integral\_value)])

The value of the integral over the interval -1 to 3 is: **-556.1969**

**Problem 3:**

syms t y(t);

dydt = diff(y, t) == 5\*cos(6\*t) + 20;

% Define the initial condition

y0 = y(0) == 40;

% Solve the differential equation

sol = dsolve(dydt, y0);

% Evaluate at t=10

t0 = 10;

y\_t = subs(sol, t, t0);

y\_t = double(y\_t);

% Display the result

disp(['The value of y at t=10 is: ' num2str(y\_t)]);

The value of y at t=10 is: **239.746**

**Problem 4:**

syms t y(t)

eq = diff(y, t, t) + 5\*diff(y, t) + 6\*y == 5\*cos(6\*t) + 20;

dy = diff(y, t);

y0 = y(0) == 20;

dy0 = dy(0) == 0;

% Solve the differential equation

y = dsolve(eq, [y0, dy0]);

% Evaluate y at t = 10

t\_value = 10;

y\_value = subs(y, t, t\_value);

y\_value = double(y\_value);

disp(['The value of y for t = 10 is: ', num2str(y\_value)]);

The value of y for t = 10 is: **3.3873**

**Problem 5:**

% Define the function

x = linspace(0, 5, 21); % Case (a): 21 points

y = 2 \* exp(-x) .\* sin(3 \* x.^2);

% Calculate the exact derivative

exact\_derivative = -2 \* exp(-x) .\* (sin(3 \* x.^2) - 6 \* x.^2 .\* cos(3 \* x.^2));

% Calculate the approximate derivative

h = x(2) - x(1);

approx\_derivative = diff(y) / h;

% Exact value of y' for x = 2.25

x\_exact = 2.25;

exact\_derivative\_2\_25 = -2 \* exp(-x\_exact) \* (sin(3 \* x\_exact^2) - 6 \* x\_exact^2 \* cos(3 \* x\_exact^2));

% Approximate value of y' for x = 2.25 with N = 21

index\_2\_25 = find(x <= x\_exact, 1, 'last');

approx\_derivative\_2\_25\_21 = approx\_derivative(index\_2\_25);

figure;

% Subplot for case (a)

subplot(1, 2, 1);

plot(x, exact\_derivative, 'b', x(1:end-1), approx\_derivative, 'r');

title('21 Points');

legend('Exact Derivative', 'Approximate Derivative');

xlabel('x');

ylabel('Derivative');

% Define the function with 501 points

x = linspace(0, 5, 501); % Case (b): 501 points

y = 2 \* exp(-x) .\* sin(3 \* x.^2);

% Calculate the exact derivative

exact\_derivative = -2 \* exp(-x) .\* (sin(3 \* x.^2) - 6 \* x.^2 .\* cos(3 \* x.^2));

% Calculate the approximate derivative

h = x(2) - x(1);

approx\_derivative = diff(y) / h;

% Subplot for case (b)

subplot(1, 2, 2);

plot(x, exact\_derivative, 'b', x(1:end-1), approx\_derivative, 'r');

title('501 Points');

legend('Exact Derivative', 'Approximate Derivative');

xlabel('x');

ylabel('Derivative');

% Exact value of y' for x = 2.3

x\_exact = 2.3;

exact\_derivative\_2\_3 = -2 \* exp(-x\_exact) \* (sin(3 \* x\_exact^2) - 6 \* x\_exact^2 \* cos(3 \* x\_exact^2));

% Approximate value of y' for x = 2.3 with N = 501

index\_2\_3 = find(x <= x\_exact, 1, 'last');

approx\_derivative\_2\_3\_501 = approx\_derivative(index\_2\_3);

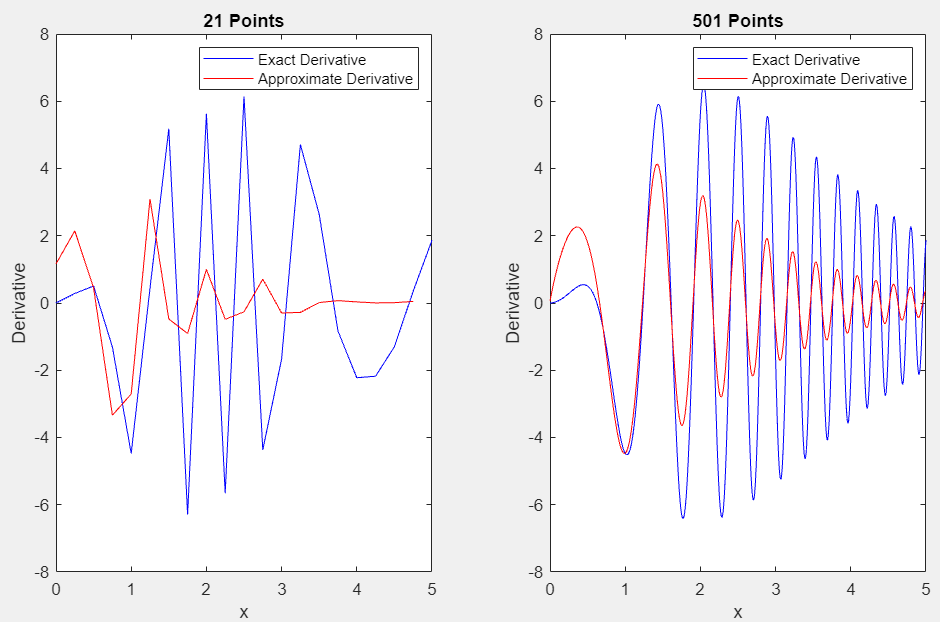
% Display the results

disp(['Exact value of y'' for x = 2.25: ', num2str(exact\_derivative\_2\_25)]);

disp(['Approx value of y'' for x = 2.25 with N = 21: ', num2str(approx\_derivative\_2\_25\_21)]);

disp(['Exact value of y'' for x = 2.3: ', num2str(exact\_derivative\_2\_3)]);

disp(['Approx value of y'' for x = 2.3 with N = 501: ', num2str(approx\_derivative\_2\_3\_501)]);



Exact value of y' for x = 2.25: **-2.5738**

Approx value of y' for x = 2.25 with N = 21: **-0.4846**

Exact value of y' for x = 2.3: **-2.6985**

Approx value of y' for x = 2.3 with N = 501: **-2.6382**

**Problem 6:**

syms x;

f(x) = 10 \* exp(-x/2) \* (sin(5\*x))^2;

% (a) Exact integral using symbolic integration

exact\_integral = int(f, 0, 10);

exact\_integral = double(exact\_integral);

% (b) Zero-order approximation

step = 0.01;

x\_values = 0:step:10;

b\_integral = sum(f(x\_values)) \* step;

b\_integral = double(b\_integral);

% (c) First-order approximation

c\_integral = trapz(x\_values, f(x\_values));

c\_integral = double(c\_integral);

% Plot the exact and approximate integrals

figure;

plot(x\_values, cumsum(f(x\_values) \* step), 'r', 'LineWidth', 2);

hold on;

plot([0, 10], [0, exact\_integral], 'b--', 'LineWidth', 2);

legend('Approximate Integral', 'Exact Integral', 'Location', 'Best');

xlabel('x');

ylabel('Integral Value');

title('Exact and Approximate Integrals');

% Display the results

disp(['Exact value of the integral for 0 to 10: ', num2str(exact\_integral)]);

disp(['Approximate value of the integral for 0 to 10 (Zero-order Approximation): ', num2str(b\_integral)]);

disp(['Approximate value of the integral for 0 to 10 (First-order Approximation): ', num2str(c\_integral)]);

Exact value of the integral for 0 to 10: **9.9095**

Approximate value of the integral for 0 to 10 (Zero-order Approximation): **9.9096**

Approximate value of the integral for 0 to 10 (First-order Approximation): **9.9095**

